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INFLUENCE OF PERIODIC TEMPERATURE ON TRANSIENT NATURAL CONVECTION FLOW OF AN ELASTICO-VISCOUS FLUID PAST A UNIFORMLY ACCELERATED PLATE

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An investigation of free convection flow of an elastico-viscous fluid past a uniformly accelerated plate with periodic temperature variation has been carried out. The solutions in a closed form have been derived for temperature, velocity, penetration distance, rate of heat transfer, and skin friction by the Laplace transform technique. A parametric study of all involved parameters is conducted and representative results are illustrated.

Keywords: *Free convection; oscillating temperature; elastico-viscous fluid.*

Introduction. The phenomenon of free convection arises in fluids, when temperature changes cause density variations that lead to the appearance of buoyancy forces acting on the fluid elements. Natural convection from a vertical plate is important in many practical applications, such as furnaces, electronic components, solar collectors, chemical processing equipments, nuclear waste materials, etc. In a practical situation, the start up and shut down of industrial devices involve consideration of transient free convection. The transition from conduction to convection begins only when some effect from the leading edge has propagated along the plate as a wave to a particular point in question. Before this time, the fluid in this region effectively "does not know" that the plate has a leading edge. Hence, Gebhart et al. introduced the idea of the leading edge effect in their book [1]. A change in the wall temperature causing the free convection flow could be sudden or periodic, leading to a variation in the flow. Further, an oscillatory flow has applications in industrial and aerospace engineering. Fluids that exhibit a nonlinear relationship between the shear stress and shear rate are said to be non-Newtonian. A number of industrially important fluids (such as polymeric materials and petroleum products), including fossil fuels which may saturate underground beds, display non-Newtonian behavior. In recent years ever increasing industrial applications in the manufacture of plastic film and artificial fiber materials have led to a renewed interest in the study of elastico-viscous fluid past a plate.

The literature is rich with investigations dealing with free convection flow past a plate. Martynenko et al. [2] studied the free convection flow of incompressible fluid past a vertical surface. Revankar [3] and Muthucumarsawamy [4] pioneered in natural convection effects on flow past a moving plate. Other studies dealing with natural convection flow (Ramanaiah et al. [5], Weiss et al. [6], and Pantokratoras [7]) included various physical aspects. In all the studies cited, the plate was assumed to be maintained at a constant temperature which is also the temperature of the surrounding stationary fluid. However, in industrial applications, it is quite often that the plate temperature starts to oscillate about a nonzero mean temperature. The flow past a surface with oscillating temperature was elucidated by Takhar et al. [8], Li et al. [9], and Saeid [10]. A more difficult problem of transient free convection flow past an isothermal plate was first studied by Siegel [11] using an integral method. The experimental confirmation of these results was presented by Goldstein et al. [12]. Another review of transient convection was given by Raithby et al. [13], who considered a large number of papers on this topic. In this review, the meaning of transient convection has been explained systematically. The authors of [13] have defined the conduction steady-state regimes and the regime which lies between these two regimes as the transient one. Later on, numerous investigators considered transient convective flow (Harris et al. [14], Das et al. [15], and Padet [16]).

All the above studies were confined to Newtonian fluids. The boundary-layer treatment for an idealized elastico-viscous fluid was carried out by Beard et al. [17]. Unsteady natural convective flow of an elastico-viscous fluid was studied by Soundalgekar [18]. Free convection flow past a surface for a non-Newtonian fluid (such as the Walters

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liquid B) was analyzed by Soundalgekar [19]. Rajagopal [20] elucidated the Stokes problem for a non-Newtonian fluid. Flow past a uniformly accelerated plate in an elastico-viscous fluid was studied by Soundalgekar [21] using the Laplace-transform technique. Teipel [22] considered the flow of the same fluid past an impulsively started plate. Similarity solution for non-Newtonian fluids was analyzed by Lee et al. [23].

The aim of this paper is to describe the transient free convection flow of an elastico-viscous fluid past a uniformly accelerated infinite vertical plate.

Mathematical Analysis. The constitutive equations for the rheological elastico-viscous fluid (Walters liquid B) are

$$p_{ik} = -p g_{ik} + p'_{ik} , \tag{1}$$

$$p'_{ik}(x, t) = 2\int_{-\infty}^{t} \psi(t - t')\, e_{ik}^{(1)}(t')\, dt' , \tag{2}$$

where

$$\psi(t - t') = \int_{0}^{\infty} \frac{N(\tau)}{\tau} \exp\left[\frac{-(t - t')}{\tau}\right] d\tau .$$

It was shown by Walters [24] that Eq. (2) can be put in the following generalized form which is valid for all types of motion:

$$p'_{ik}(x, t) = 2\int_{-\infty}^{t} \psi(t - t') \frac{\partial x_i}{\partial x'_m} \frac{\partial x_k}{\partial x'_r} e_{mr}^{(1)}(x', t')\, dt' , \tag{3}$$

where $x'_i = x'_i(x, t, t')$ is the position at time t' of the element which is located instantaneously at time t at the point x_i.

The fluid with equations of state (1) and (3) has been designated as liquid B′. In the case of liquids with short memories (i.e., with short relaxation times), the above equation of state can be written in the following simplified form:

$$p'_{ik}(x, t) = 2\eta_0 e_{ik}^{(1)} - 2k_0 \frac{\partial e_{ik}^{(1)}}{\partial t} , \tag{4}$$

where

$$\eta_0 = \int_{0}^{\infty} N(\tau)\, d\tau$$

is the limiting viscosity at small rates of shear,

$$k_0 = \int_{0}^{\infty} \tau N(\tau)\, d\tau ,$$

and $\frac{\partial}{\partial t}$ denotes the convected time derivative.

Hence, we consider the flow of an elastico-viscous fluid past an infinite vertical plate with x' axis along the plate in the vertical upward direction and y' axis normal to it. As the surface is infinite, the physical variables are functions of y and t only. Initially, the plate and fluid are at the same temperature T'_∞. At time $t' > 0$, the plate temperature is raised to T'_w and a periodic temperature is assumed to be superimposed on this mean constant temperature of the plate that accelerates with a velocity $U_r^3 t'/\nu$. Then with neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq approximation), the flow and temperature field are governed by the following equations:

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2}, \tag{5}$$

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{k_0}{\rho} \frac{\partial^3 u'}{\partial y'^2 \partial t'} + g\beta (T' - T'_\infty) \tag{6}$$

with the following initial and boundary conditions:

$$u' = 0, \quad T' = T'_\infty \quad \text{for} \quad \forall y', t' \le 0,$$

$$u' = \frac{U_r^3}{\nu} t', \quad T' = T'_w + A \left(T'_w - T'_\infty\right) \cos \omega' t' \quad \text{at} \quad y' = 0, \quad t' > 0,$$

$$u' \to 0, \quad T' \to T'_\infty, \quad \text{as} \quad y' \to \infty, \quad t' > 0. \tag{7}$$

The temperature distribution is independent of the flow, and heat transfer is carried out by conduction alone. This is true for fluids in the initial stage due to the absence of convective heat transfer or at small Grashof number.

We introduce the following nondimensional quantities in Eqs. (5)–(7):

$$t = \frac{t'}{t_r}, \quad y = \frac{y'}{L_r}, \quad u = \frac{u'}{U_r}, \quad \omega = \omega' t_r,$$

$$\Pr = \frac{\mu C_p}{\kappa}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \Delta T = T'_w - T'_\infty,$$

$$k = \frac{k_0 U_r^2}{\rho \nu^2}, \quad U_r = (\nu g \beta \Delta T)^{1/3}, \quad L_r = \left(\frac{g\beta\Delta T}{\nu^2}\right)^{-1/3},$$

$$t_r = (g\beta\Delta T)^{-2/3} \nu^{1/3}. \tag{8}$$

Equations (5)–(7) are reduced to nondimensional form

$$\Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2}, \tag{9}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta - k \frac{\partial^3 u}{\partial y^2 \partial t} \tag{10}$$

with the following initial and boundary conditions:

$$u = 0, \quad \theta = 0 \quad \text{for} \quad \forall y, t \le 0,$$

$$u = t\,,\quad \theta = 1 + A\cos\omega t \quad \text{at} \quad y = 0,\, t > 0\,,$$

$$u \to 0\,,\quad \theta \to 0\,,\quad \text{as} \quad y \to \infty\,,\quad t > 0\,. \tag{11}$$

Equation (10) is a third-order differential equation to be solved with the boundary conditions (11). To solve it, we expand u in powers of k as follows:

$$u = u_1 + k u_2 + O\,(k^2)\,. \tag{12}$$

Substituting (12) in (10), equating the coefficients at different powers of k, and neglecting the terms of the order of k^2, we get

$$\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_1}{\partial y^2} + \theta\,, \tag{13}$$

$$\frac{\partial u_2}{\partial t} = \frac{\partial^2 u_2}{\partial y^2} - \frac{\partial^4 u_1}{\partial y^4} - \frac{\partial^2 \theta}{\partial y^2} \tag{14}$$

and the boundary conditions

$$u_1 = t\,,\quad u_2 = 0 \quad \text{at} \quad y = 0\,, \tag{15}$$

whereas the other conditions remain the same.

The linear equations (9), (13), and (14) are solved in a closed form by the usual Laplace-transform technique, and the solutions are as follows:

$$\theta = \operatorname{erfc}\,(\eta\sqrt{\Pr}) + \frac{A}{2}\left\{g(\eta\sqrt{\Pr},\, i\omega) + g(\eta\sqrt{\Pr},\, -i\omega)\right\}, \tag{16}$$

for $\Pr \neq 1$

$$U = \frac{u_1 + k u_2}{t} = \frac{\Pr k}{(\Pr - 1)^2\, t}\left\{\operatorname{erfc}\,\eta - \operatorname{erfc}\,(\eta\sqrt{\Pr})\right\}$$

$$- \frac{\eta\,(\Pr + A)\, k \exp\,(-\eta^2)}{\sqrt{\pi}\,(\Pr - 1)\, t} + \frac{1}{\Pr - 1}\left\{\Pr f\,(\eta) - f\,(\eta\sqrt{\Pr})\right\}$$

$$- \frac{\exp\,(i\omega t)}{t}\left\{\frac{iA}{4\omega\,(\Pr - 1)} - \frac{\Pr kA}{4\,(\Pr - 1)^2}\right\}\left\{g(\eta,\, i\omega) - g(\eta\sqrt{\Pr},\, i\omega)\right\}$$

$$+ \frac{\exp\,(-i\omega t)}{t}\left\{\frac{iA}{4\omega\,(\Pr - 1)} + \frac{\Pr kA}{4\,(\Pr - 1)^2}\right\}\left\{g(\eta,\, -i\omega) - g(\eta\sqrt{\Pr},\, -i\omega)\right\}$$

$$+ \frac{\eta k}{4\,(\Pr - 1)\sqrt{t}}\left\{m\,(\eta,\, i\omega) + m\,(\eta,\, -i\omega)\right\}, \tag{17}$$

where

$$\eta = \frac{y}{2\sqrt{t}}\,,$$

$$h(a) = \frac{t^2}{2} f(a) - \frac{at^2}{3} \left\{ \frac{2a^2 \exp(-a^2)}{\sqrt{\pi}} + \frac{2\exp(-a^2)}{\sqrt{\pi}} - a(1 + 2a^2) \operatorname{erfc} a - 2a \operatorname{erfc} a \right\},$$

$$f(a) = (1 + 2a^2) \operatorname{erfc} a - \frac{2a \exp(-a^2)}{\sqrt{\pi}},$$

$$g(a, b) = \exp(2a\sqrt{bt}) \operatorname{erfc}(a + \sqrt{bt}) + \exp(-2a\sqrt{bt}) \operatorname{erfc}(a - \sqrt{bt}),$$

$$m(a, b) = \sqrt{b} \exp(bt) \left\{ \exp(2a\sqrt{bt}) \operatorname{erfc}(a + \sqrt{bt}) - \exp(-2a\sqrt{bt}) \operatorname{erfc}(a - \sqrt{bt}) \right\},$$

$$a = \eta \quad \text{or} \quad \eta\sqrt{\Pr}, \quad b = i\omega \quad \text{or} \quad -i\omega.$$

Initially, heat is transferred to the plate by conduction but at a slightly later stage convection currents start flowing near the plate. Hence, it is essential to know the position of a point on the plate, where the conduction mechanism changes to the convection one. In literature, this is known as the leading edge effect. The distance of this point of transition from conduction to convection is given by

$$X_p = \int_0^t u_3(y, t)\, dt \tag{18}$$

or, in terms of the Laplace transform and its inverse,

$$X_p = L^{-1}\left[\frac{1}{p} L\left\{ u_3(y, t) \right\} \right],$$

where p is the Laplace transformation parameter and $u_3 = u_1 + ku_2$.

Substituting (17) for $L\{u_3(y, t)\}$ and carrying out the simplification, we have for $\Pr \neq 1$

$$X_p = \frac{1}{\Pr - 1} \left\{ \Pr h(\eta) - h(\eta\sqrt{\Pr}) \right\} + \frac{\Pr kt}{(\Pr - 1)^2} \left\{ f(\eta) - f(\eta\sqrt{\Pr}) \right\}$$

$$- \left(\frac{A}{\omega} + \frac{ik \Pr A}{\Pr - 1} \right) \frac{\exp(i\omega t)}{4\omega(\Pr - 1)} \left\{ g(\eta, i\omega) - g(\eta\sqrt{\Pr}, i\omega) \right\}$$

$$- \left(\frac{A}{\omega} - \frac{ik \Pr A}{\Pr - 1} \right) \frac{\exp(-i\omega t)}{4\omega(\Pr - 1)} \left\{ g(\eta, -i\omega) - g(\eta\sqrt{\Pr}, -i\omega) \right\}$$

$$+ \frac{Ai\eta k\sqrt{t}}{4\omega(\Pr - 1)} \left\{ m(\eta, -i\omega) - m(\eta, i\omega) \right\}$$

$$- \frac{2\eta \Pr tk}{\Pr - 1} \left\{ \frac{1}{\sqrt{\pi}} \exp(-\eta^2) - \eta \operatorname{erfc} \eta \right\} + \frac{A}{\omega^2 (\Pr - 1)} \left\{ \operatorname{erfc} \eta - \operatorname{erfc}(\eta\sqrt{\Pr}) \right\}. \tag{19}$$

In the expressions above, $\operatorname{erfc}(x_1 + iy_1)$ is the complementary error function of the complex argument, which can be calculated in terms of tabulated functions [25]. The table in [25] gives not $\operatorname{erfc}(x_1 + iy_1)$ directly but only an auxiliary function $W_1(x_1 + iy_1)$ which is defined as

$$\operatorname{erfc}(x_1 + iy_1) = W_1(-y_1 + ix_1) \exp\left\{ -(x_1 + iy_1)^2 \right\}.$$

Some properties of $W_1(x_1 + iy_1)$ are the following:

$$W_1(-x_1 + iy_1) = W_2(x_1 + iy_1),$$

$$W_1(x_1 - iy_1) = 2\exp\left\{-(x_1 - iy_1)^2\right\} - W_2(x_1 + iy_1),$$

where $W_2(x_1 + iy_1)$ is the complex conjugate of $W_1(x_1 + iy_1)$.

Skin friction. From the velocity field, the skin friction is given as

$$\tau' = \left(\mu \frac{\partial u'}{\partial y'} - k_0 \frac{\partial^2 u'}{\partial y' \partial t'}\right)_{y'=0}. \quad (20)$$

In view of (8), this reduces to

$$\tau = \left(\frac{\partial u}{\partial y} - k \frac{\partial^2 u}{\partial y \partial t}\right)_{y=0}. \quad (21)$$

Substituting (12) in (21) and neglecting coefficient of the order of k^2, we get

$$\tau = \left.\frac{\partial u_1}{\partial y}\right|_{y=0} + k\left(\frac{\partial u_2}{\partial y} - \frac{\partial^2 u_1}{\partial y \partial t}\right)_{y=0}. \quad (22)$$

For $\Pr \neq 1$

$$\tau = \frac{\Pr k(\sqrt{\Pr} - 1)}{(\Pr - 1)^2 \sqrt{t\pi}} - \frac{(\Pr + A)k}{2(\Pr - 1)\sqrt{\pi t}} + \frac{2\sqrt{\Pr t}}{(\Pr - 1)\sqrt{\pi}}(1 - \sqrt{\Pr})$$

$$+ \frac{\kappa(1 - \sqrt{\Pr})^2}{(\Pr - 1)\sqrt{\pi t}} + \exp(i\omega t)\left(\frac{iA}{2\omega(\Pr - 1)} - \frac{\Pr kA}{2(\Pr - 1)^2}\right)$$

$$+ \left\{q(i\omega) - \sqrt{\Pr}\, q(i\omega)\right\} - \exp(-i\omega t)\left(\frac{iA}{2\omega(\Pr - 1)} + \frac{\Pr kA}{2(\Pr - 1)^2}\right)$$

$$+ \left\{q(-i\omega) - \sqrt{\Pr}\, q(-i\omega)\right\} + \frac{Ak\sqrt{i\omega}\exp(i\omega t)\operatorname{erf}(\sqrt{i\omega t})(0.5 - \sqrt{\Pr})}{2(\Pr - 1)}$$

$$+ \frac{Ak\sqrt{-i\omega}\exp(-i\omega t)\operatorname{erf}(\sqrt{-i\omega t})(0.5 - \sqrt{\Pr})}{2(\Pr - 1)}. \quad (23)$$

Nusselt number. From the temperature field, the rate of heat transfer in nondimensional form is expressed as

$$\mathrm{Nu} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}, \quad (24)$$

$$\mathrm{Nu} = \sqrt{\frac{\Pr}{\pi t}} + \frac{A\sqrt{\Pr}}{2}\left\{\exp(-i\omega t)\, q(-i\omega) + \exp(i\omega)\, q(i\omega)\right\}, \quad (25)$$

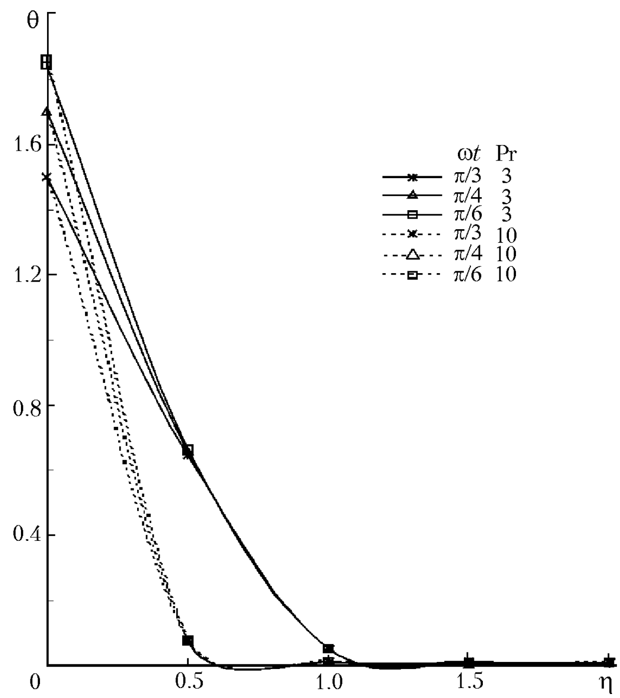

Fig. 1. Transient temperature profiles for different values of the phase angle ωt and Prandtl number Pr.

where

$$q(d) = \sqrt{d}\ \mathrm{erf}\sqrt{dt} + \frac{1}{\sqrt{\pi t}} \exp(-dt), \quad d = i\omega \ \text{ or } \ -i\omega .$$

Discussion. In order to get an insight into the physical situation of the problem, we have computed the values of temperature, velocity, penetration distance, Nusselt number, and skin friction for different values of Prandtl number, non-Newtonian parameter, phase angle, and time. The values of the Prandtl number, which are taken as 3 and 10, represent the saturated liquid (refrigerant CCl_2F_2) at 272.3 K and gasoline at atmospheric pressure and 20°C, respectively. The value of A is taken equal to unity.

Figure 1 reveals the transient temperature profiles against η (distance from the plate) for different values of phase angle ωt and Prandtl number Pr. It is evident from the figure that the temperature is maximum at the plate, then decreases, and away from the plate tends to zero. It decreases with an increase in ωt when Pr is fixed. A comparative study of the curves indicates that for a fixed value of ωt the temperature decreases with an increasing Pr. This is in agreement with the physical fact that the thermal boundary-layer thickness decreases with an increasing Pr. The figure also shows that the decrease in the magnitude of temperature for gasoline is sharper than for the refrigerant CCl_2F_2. The velocity and penetration distance for different values of ωt and Pr are plotted in Figs. 2 and 3, respectively. The analysis of these figures reveals that both the velocity and penetration are maximum at the surface and then fall sharply. Further, the magnitudes of both the velocity and penetration for Pr = 3 are higher than those for Pr = 10. This is due to the fact that fluids with a high Prandtl number have high viscosity and hence move slowly. These figures also indicate that with increase in ωt both the velocity and penetration remain unchanged for gasoline and change slightly for CCl_2F_2. The point of separation takes place for different values of the parameters ωt and Pr.

Figures 4 and 5 illustrate the influences of k, t, and Pr on the velocity and penetration, respectively. It is seen from these figures that both the velocity and penetration are maximum at the surface, then decrease and move asymptotically. It is inferred from the figures that the magnitude of both quantities mentioned decrease with an increase in the non-Newtonian parameter k for both fluids. For a Newtonian fluid (k = 0), the velocity and penetration are greater in the whole flow field, when the time is fixed. On the other hand, they increase with time for a non-Newtonian fluid.

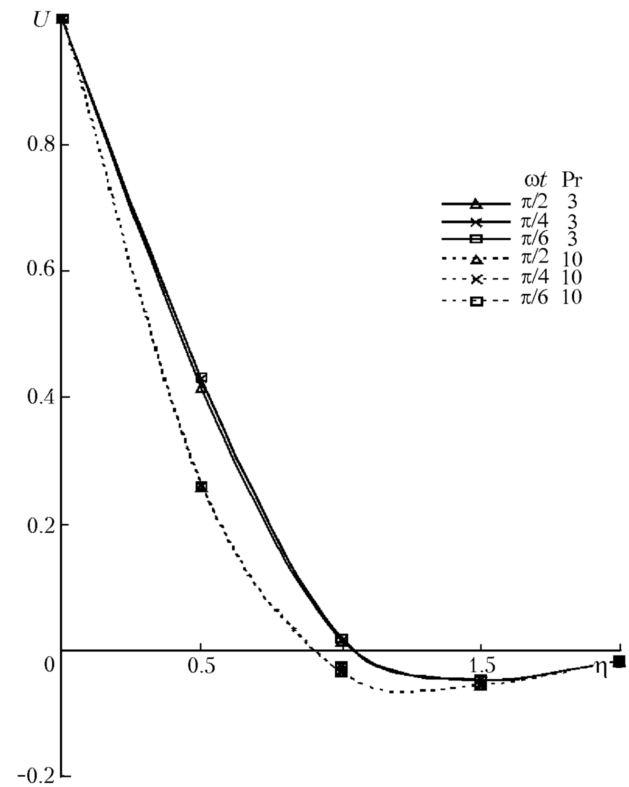

Fig. 2. Velocity profiles at k = 0.1 and t = 0.2 for different values of the phase angle ωt and Prandtl number Pr.

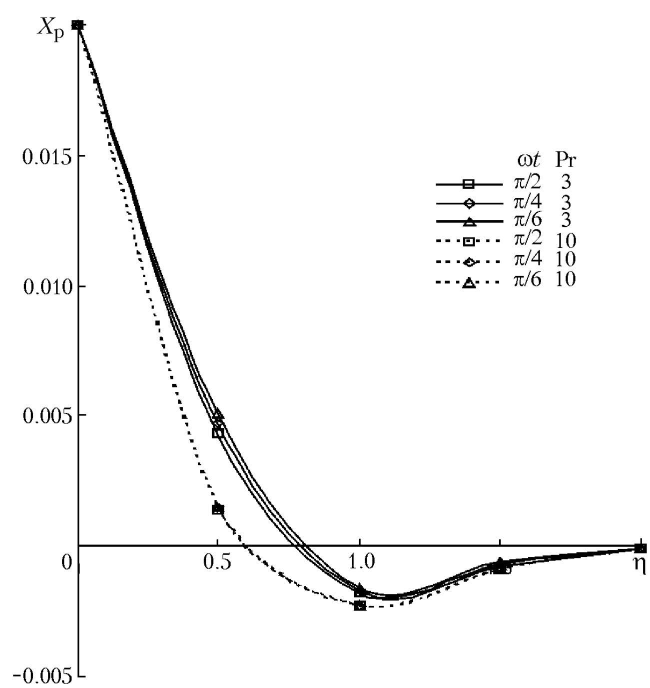

Fig. 3. Profiles of the penetration distance at k = 0.1 and t = 0.2 for different values of the phase angle ωt and Prandtl number Pr.

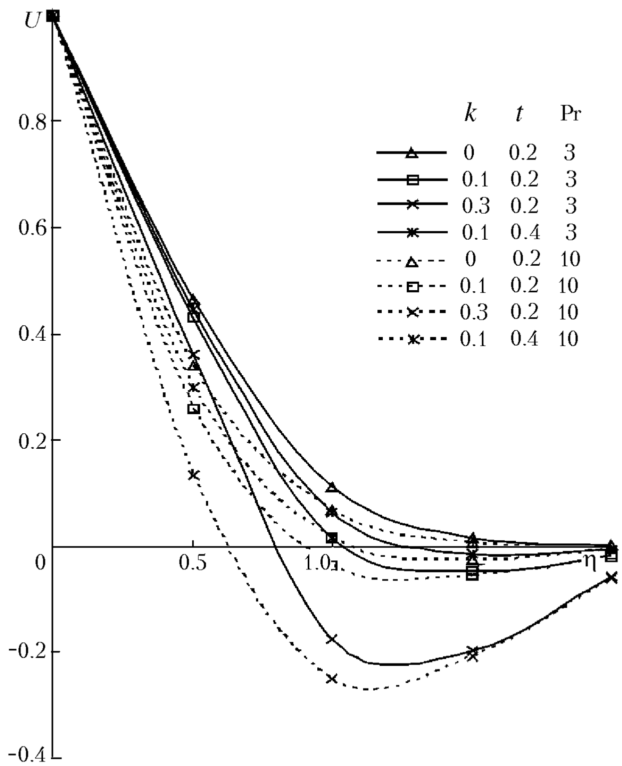

Fig. 4. Velocity profiles at $\omega t = \pi/4$ for different values of the elastic parameter k, time t, and Prandtl number Pr.

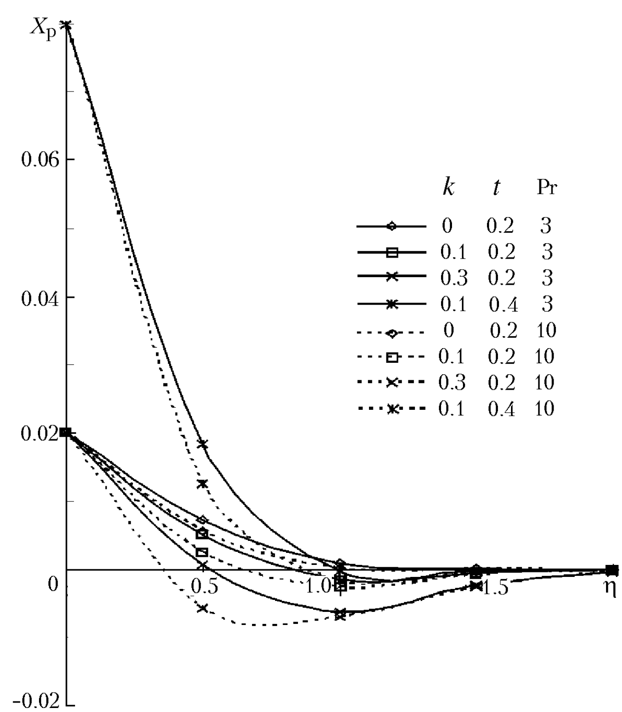

Fig. 5. Profiles of the penetration distance at $\omega t = \pi/4$ for different values of the elastic parameter k, time t, and Prandtl number Pr.

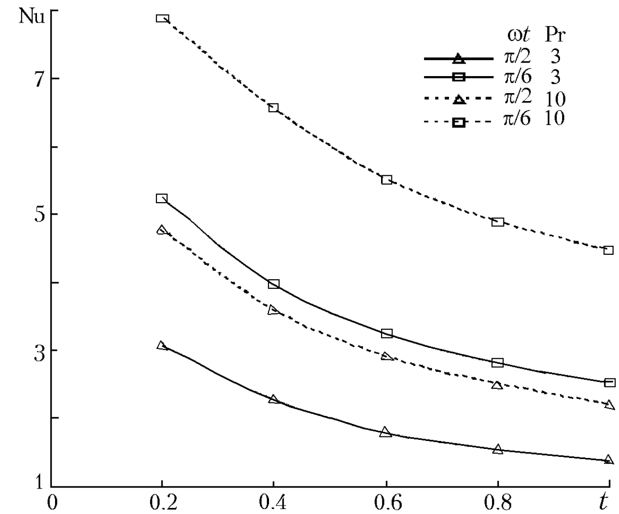

Fig. 6. Nusselt number vs. time for different values of the phase angle ωt and Prandtl number Pr.

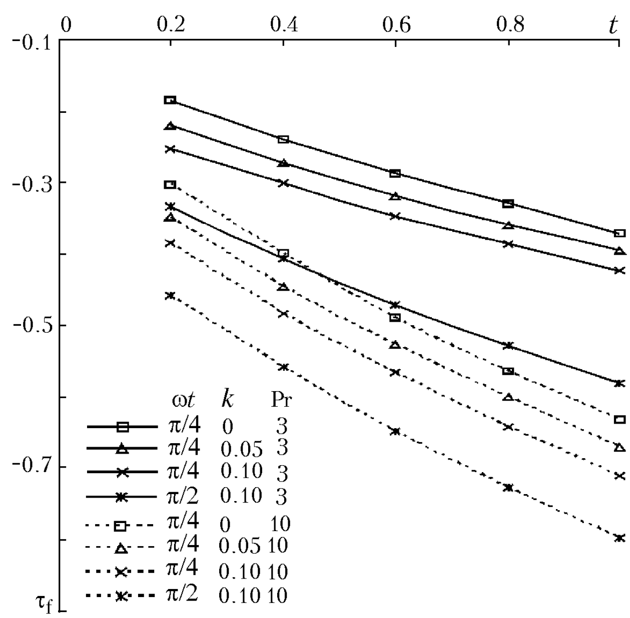

Fig. 7. Skin friction vs. time for different values of the phase angle ωt and Prandtl number Pr.

The rate of heat transfer vs. time is shown in Fig. 6. It is clear from the figure that Nu decreases with an increase in ωt for both values of Pr. The values of the Nusselt number for Pr = 10 is higher than for Pr = 3. This is due to the fact that there would be a decrease in thermal conductivity with an increase in the Prandtl number; therefore more heat is diffused for lower values of Pr. Figure 7 elucidates the skin friction for various values of ωt, k, and Pr against t. The figure reveals that the skin friction decreases with an increasing k for both refrigerant and gasoline and it is greater for the Newtonian fluid ($k = 0$) in comparison to the elastico-viscous one. It also decreases with increase in the phase angle. In addition, the magnitude of the skin friction for Pr = 3 is higher than for Pr = 10. Physically, this is true because the increase in the value of Pr is due to the increase in the viscosity of the fluid, which makes the fluid thick and leads to the decrease in its velocity.

NOTATION

A, amplitude; C_p, specific heat at constant pressure; $e_{ik}^{(1)}$, rate of strain tensor; g, gravitational acceleration; g_{ik}, metric tensor of fixed coordinate system x_i; k, elastic parameter; L_r, reference length; $N(\tau)$, distribution function over the relaxation times; Nu, Nusselt number; P, isotropic pressure; p_{ik}, stress tensor; Pr, Prandtl number; T', temperature of fluid near the plate; T'_w, plate temperature; T'_∞, temperature of fluid far away from the plate; t, time; t_r, reference time; u, dimensionless velocity component; U_r, reference velocity; X_p, distance of the transition point from the leading edge; x, y, dimensionless coordinates; β, coefficient of volume expansion; θ, dimensionless temperature; κ, thermal conductivity of fluid; μ, fluid viscosity; ν, kinematic viscosity; ρ, fluid density; τ, relaxation time; τ_f, skin friction; ω, frequency of oscillation; ωt, phase angle. Subscripts: w, wall; ∞, at infinity. The primes in symbols correspond to dimensional quantities.

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